MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

EME3156-COMPUTATIONAL METHODS FOR MECHANICAL ENGINEERING
(ME)

15 OCTOBER 2015
9.00 a.m. – 11.00 a.m..
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 4 pages excluding the cover page.

2. Answer all the questions. The distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided. All necessary workings **MUST** be shown.

Question 1

A nonlinear system of equations is given:

$$\begin{aligned} x_1^3 - x_2 &= 1 \\ -x_1 + x_2^3 &= 1 \end{aligned} \qquad (1.1)$$

Let $x = [x_1 \quad x_2 \quad]^T$ and $\overline{F}(x) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix}$ where $\overline{F}$ is a vector-value function of a vector argument **x**. The original nonlinear system can be expressed as the vector equation $F(x) = 0$. 0 is a zero vector. Newton method is used to solve the above non-linear system of equations (1.1). Given an initial approximation, x_0, for a root of that function, Newton's method computes a sequence of improved approximations to the root. In order to solve the simultaneous equations, these equations are assembled to become a matrix equations $[J]\{\Delta x\} = \{F\}$.

a) Write down the Newton method for solving nonlinear system. **[4 marks]**

b) Find the Jacobian matrix for the system, $[J(x^0)]$ and $F(x^{(0)})$ when the initial guess is given as $x^{(0)} = [3 \quad 2]^T$. **[10 marks]**

c) Determine the improved approximation $x^{(1)}$. **[11 marks]**

Continued...

Question 2

a) Equation (2.1) is a first order ordinary differential equation(ODE) with initial condition,

$$\frac{dy}{dx} + 2y - x - 4 = 0 \ , \quad y(0) = 1 \tag{2.1}$$

With step size h = 0.2 , determine $y(0.2)$ using the fourth order Runge-Kutta method, RK4.

$$k_1 = f(x_i, y_i)$$
$$k_2 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}k_1)$$
$$k_3 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}k_2)$$
$$k_4 = f(x_i + h, y_i + hk_3)$$
$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

[13 marks]

Compare your result with exact solution, given that the exact solution is $y = -0.75e^{-2x} + 0.5x + 1.75$.

b) A second order ODE with boundary conditions is given as

$$\frac{d^2y}{dx^2} + y = 0 \ , \quad y(0) = 1, \ y(\frac{\pi}{2}) = 0 \tag{2.2}$$

(i) Equation (2.2) is converted into two first order ODE with one assumption of condition. Write down the two ODEs with the conditions. The value of the assumption is set to zero. Use 'u' and 'v' as the new variables. **[4 marks]**

(ii) Shooting method is used to solve equation (2.2). Use Explicit Euler method with step size $h = \frac{\pi}{8}$, find $y(\frac{\pi}{2})$. **[8 marks]**

Continued...

Question 3

The governing equation of a one-dimensional, transient heat conduction problem is given in equation (3.1).

$$u_t = \alpha u_{xx} + Q \tag{3.1}$$

u is the temperature measured along a one-dimensional system. The constant α is the thermal diffusivity, the ratio of heat conduction to the heat storage. The independent variables are x and t. For the problem to be solved, $\alpha = 1$ and $Q = 5$. The domain of the system is 1.0 m long and time interval $\Delta t = 0.01$ s is used. The initial and boundary conditions are given by

$$\begin{cases} u(x,0) = 30 & 0 \le x \le 1.0 \\ u(0,t) = 50 & t > 0 \\ u(1.0,t) = 30 & t \ge 0 \end{cases}$$

a) Convert equation (3.1) into explicit finite difference form using central difference scheme. **[5 marks]**

b) Write down the finite difference form as obtained in (a) in matrix form, (3.2).

$$\{u\}^{t+1} = [A]\{u\}^t + \{BC\} \tag{3.2}$$

$\{u\}^{t+1}$ and $\{u\}^t$ are vectors for nodal variables at time t+1 and t respectively. $[A]$ is a square matrix. The bar (space domain) is discretised into six nodes. Initial and boundary conditions $\{BC\}$ must be incorporated into the matrix equation.

[12 marks]

c) Solve for $\{u\}^1$, vector for nodal variables at time t+1 . **[8 marks]**

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Question 4

A problem of two-dimensional, steady state heat conduction with no heat source, with boundary conditions included is illustrated in Fig. 4.1. Node 1 is the origin of the coordinate system.

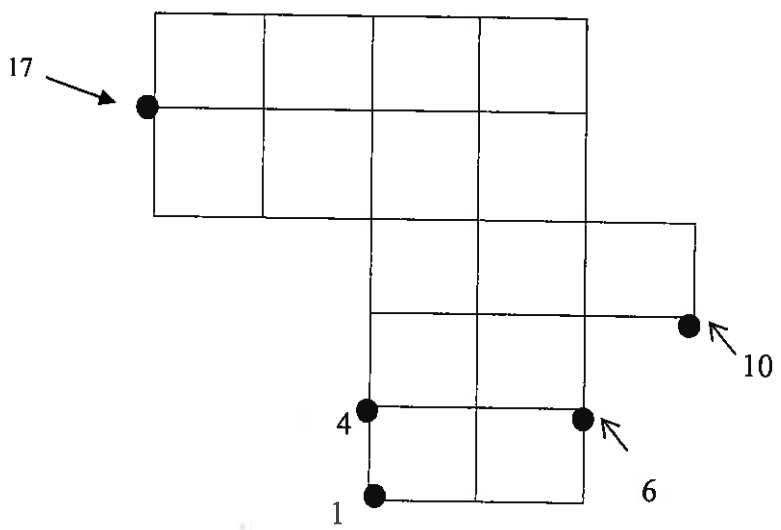

Fig 4.1

Fig. 4.1 above shows a plate consists of 15 rectangles with each of them of the size 1cm x 1cm. The labeling of the nodes start from left to right and then goes from bottom to top.

Node 1, 2, 3, 6, 9, 10, 15 and 16 are at temperature 70°C
Node 11,17, 21, 22, 23, 24, 25 and 26 are at temperature 60°C
Node 4, 7, 12, 13 are insulated

The finite difference equation used to model the discretization is shown in the equation (4.1)

$$T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1} - 4T_{i,j} = 0 \qquad (4.1)$$

a) Copy Fig. 4.1 into your answer sheet, and label all the missing nodes.
[2 marks]

b) Write down all the finite difference equations of the domain by using equation (4.1). You must incorporate the boundary conditions into these equations where applicable.
[13 marks]

c) Assemble all the equations above to rewrite them in matrix form. **[10 marks]**

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